



Durational Patterning at Discourse Boundaries in Relation to Therapist Empathy in Psychotherapy

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Abstract

This study investigates how durational patterning at discourse boundaries (duration of utterance-final syllables, silent pause, and speech rate) is related to therapist empathy in psychotherapy. Four psychotherapy sessions each from 39 therapist-client dyads conducted in Cantonese were videotaped, transcribed, and analyzed. Clients rated therapist empathy using the Barrett-Lennard Relationship Inventory (BLRI). Mixed-effects regression showed significant effects of silent pause and speech rate on BLRI. The shorter the silent pause or the faster the speech rate, the higher the therapist empathy. Additionally, there were significant interaction effects of the duration of utterance-final syllables with silent pause and speech rate respectively. For the same unit of increment in the duration of utterance-final syllables, shorter silent pause or faster speech rate was predicted to have a greater magnitude of improvement in therapist empathy than longer silent pause or slower speech rate. Also, the interaction between silent pause and speech rate was significant. For utterances with fast (slow) speech rates, the longer the silent pause, the lower (higher) the therapist empathy. Our results have shown that clients integrated low-order durational patterning at the discourse boundaries in their higher-order perception of therapist empathy, which have clinical/educational implications for the use of prosody in psychotherapy.

Index Terms: durational patterning, prosodic phrasing, discourse boundaries, therapist empathy, psychotherapy

1. Introduction

This study investigates the relationships between durational patterning at discourse boundaries and client ratings of therapist empathy in the speech of Cantonese psychotherapy.

1.1. Therapist empathy

Clients' experience of therapist empathy has long been hypothesized to be a key process in psychotherapy contributing to client change [1], [2]. It can be defined broadly as "the therapist's sensitive ability and willingness to understand the client's thoughts, feelings and struggles from the client's point of view" [3]. Empathy is a higher-order process of human emotions; however, little is known about its low-order prosodic components, even though prosody had been shown to contribute to the perception of empathy [4]. In the recent

decade, scholars have started to explore the relationships between prosody and therapist empathy, particularly focusing on the pitch and intensity aspects of intonation [5]–[9]. While the pitch dimension of prosodic phrasing (e.g., use of intonation, pitch level, and pitch span) is associated with therapist empathy [9], to the best of our knowledge, there is no previous study specifically investigating the temporal dimension of prosodic phrasing. Discourse boundaries are considered as a universal marker of prosodic phrasing [10], [11], of which final lengthening and silent pause are widely considered to be robust cues. Besides, prosodic phrasing is often influenced by speech rate. Speech rate is a temporal factor that interacts with final lengthening and silent pause at the discourse boundaries in speech production [10], [12]. It is, however, still unclear how these temporal cues of prosodic phrasing affect clients' perception of therapist empathy.

1.2. Durational patterning at discourse boundaries

Early studies showed that lengthening in English, a stress-timed language, tends to fall on sentence-final units, including the rhyme [13] and the foot [14]. Its "boundary duration" can be defined as the interval between the onset of the last stressed syllable preceding a clausal boundary and the onset of phonation of the first syllable after the boundary [15]. In contrast, the "boundary duration" of Cantonese, a syllable-timed language [16], is defined as the interval between two contiguous syllable onsets, which would include the duration of the preboundary syllable itself and any pause that potentially follows [17], [18]. An ideal Cantonese prosodic phrasing implies a clear final lengthening at clause-final syllable and silent pause. As far as the duration of utterance-final syllables is concerned, Cantonese also employs duration as a component of intonation (e.g., :%) [19]–[21] which is strictly licensed at the utterance-final syllables [19], [22], [23]. Mai [24] argued that protracted utterance-final syllables in Cantonese emphasize the sentential discourse functions, while the shortened ones intensify speakers' paralinguistic emotions. Furthermore, sentence-final particles are very commonly used in Cantonese. Sybesma and Li [25] argued that the protracted particles often smoothen emotional expressions, while the shortened ones intensify the expressed emotions. Since empathy contributes to psychotherapy by supporting and promoting clients' active self-healing and change efforts [26], therapists' strong personal emotions should be avoided. Hence, using shortened utterance-final syllables and particles is hypothesized to have a negative

effect on clients' perception of therapist empathy. On the contrary, protracted utterance-final syllables comply with prosodic phrasing with more conspicuous final lengthening. We thus hypothesize that the longer the duration of utterance-final syllables, the higher the therapist empathy.

Silent pause is found to be a robust cue for discourse unit delimitation. The length of pause can vary according to the sizes and syntactic boundaries of the "prosodic chunks" [27]–[29]. However, studies have shown that long clausal silent pauses reduce perceived willingness and affirmation and may even signal hesitation [30], [31]. We, therefore, hypothesize that long silent pauses are negatively associated with therapist empathy.

In previous research on psychotherapy, the focus was placed on the synchrony of speech rates between client and therapist [32]–[34]. The present study specifically investigates durational patterning of therapists' speech, which affects the clarity of prosodic phrasing and perceived therapist empathy. Previous literature showed that fast speech is widely stereotyped/biased as credible, trustworthy, and persuasive speech [35]–[38]. We thus hypothesize that faster speech can signify energy to clients and help them feel that therapists are helping them to move forward, and thus can be perceived with higher therapist empathy. Importantly, at fast speech rates, clear final lengthening is a crucial temporal cue of prosodic phrasing [39]. Also, with a view to Cantonese intonation, protracted utterance-final syllables sound pragmatically/paralinguistically smoother than the shortened ones. We hypothesize that if therapists speak at a higher speech rate, along with the longer utterance-final syllables, the higher the clients' perception of therapist empathy. In addition, since long clausal silent pause reduces perceived willingness and may signal hesitation, and that long pauses in fast speech sound more dysfluent than those in slow speech (the prosody/rhythm is overall steadier) [40], we hypothesize that if therapists speak at a higher speech rate, along with longer duration of silent pause, the lower the clients' perception of therapist empathy.

1.3. Present study

The current study investigates how durational patterning at discourse boundaries, namely duration of utterance-final syllables (final lengthening and protracted/shortened syllable/intonation), silent pause, and speech rate, is in relation to perceived therapist empathy in Cantonese psychotherapy. This study can bridge an important gap in our understanding of how low-order temporal cues of prosodic phrasing can contribute to the perception of therapist empathy by clients, a higher-order process of psychotherapy.

2. Methods

2.1. Data collection

39 unique therapist-client dyads were recruited. Four 50-minute psychotherapy sessions from each of the 39 dyads, i.e., a total of 156 sessions, were videotaped and voice-recorded by lavalier microphones worn by both the therapists and the clients in the sound-attenuated counseling rooms of the counseling clinic at the Department of Educational Psychology of The Chinese University of Hong Kong. All sessions were conducted

predominantly in Cantonese with occasional English code-mixing.

After each session, clients rated therapist empathy using the Barrett-Lennard Relationship Inventory (BLRI) [41], a well-established measure of therapist empathy in psychotherapy. The rating of BLRI ranges from -48 to 48. A higher rating indicates more therapist empathy perceived by the clients.

2.2. Procedure

Recordings of all psychotherapy sessions were manually transcribed. Speech data were first manually time-stamped at the turn level, followed by forced speech-text alignment to obtain the syllable-level time stamps, i.e., the beginning and ending time of each syllable. Since the current study concerns about therapist empathy as perceived by clients, the acoustics of clients' speech seem less relevant; thus, only utterances of therapists were extracted and analyzed (total number of utterances = 81,011), where utterances were delimited based on the transcription of native judgments. Following the practice in previous literature [18], [27], [42], we calculated the duration of the last syllable of each utterance, the duration of silent pause after each utterance, and the pause-excluding speech rate of each utterance (measured by dividing the number of syllables by the duration of articulation).

Mixed-effects regression was performed on the duration data using *lme4* [43] and *car* packages [44] in R [45] with BLRI as a continuous dependent variable. There were three predictors in the model: (1) syllable duration of the utterance-final syllable, (2) duration of silent pause after each utterance, and (3) speech rate of each utterance; all predictors were square root-transformed in modeling. We also included random intercepts for the psychotherapy sessions, the therapy-client dyads, the age and sex of both clients and therapists, and the variability of utterances in our model.

3. Results

Figure 1 shows the results of our mixed-effects regression using the R package *sjmisc* [46]. First, silent pause is found to have significant main effects on BLRI, $\chi^2(1) = 4.365, p = .037$. Figure 1(A) shows that as the duration of silent pause increases, the predicted value of BLRI decreases. Second, there are significant main effects of speech rate, $\chi^2(1) = 8.773, p = .003$. Figure 1(B) shows that the predicted value of BLRI increases as the speech rate increases. The main effects of duration of the utterance-final syllables are, however, found to be non-significant, $\chi^2(1) = 1.152, p = .283$.

Additionally, there are significant two-way interaction effects between the three predictors. First, despite the non-significant main effects of the duration of utterance-final syllables, its interaction effects with silent pause are found significant, $\chi^2(1) = 5.260, p = .022$. Figure 1(C) shows that the predicted value of BLRI is generally higher as the duration of utterance-final syllables increases for all types of silent pause duration (long, average, and short). However, the slope of utterances with short silent pauses (red line, pause duration 1 SD below mean)¹ is steeper than that of utterances with average (blue line, pause duration at mean) and long silent pauses (green

¹ We obtained a negative value (-.04 sqrt s) based on the prediction of our regression model by subtracting 1 SD (.32 sqrt s) from the mean (.28 sqrt s). It is neither possible to have a "negative pause" in reality, nor did we have a negative value in the observed data.

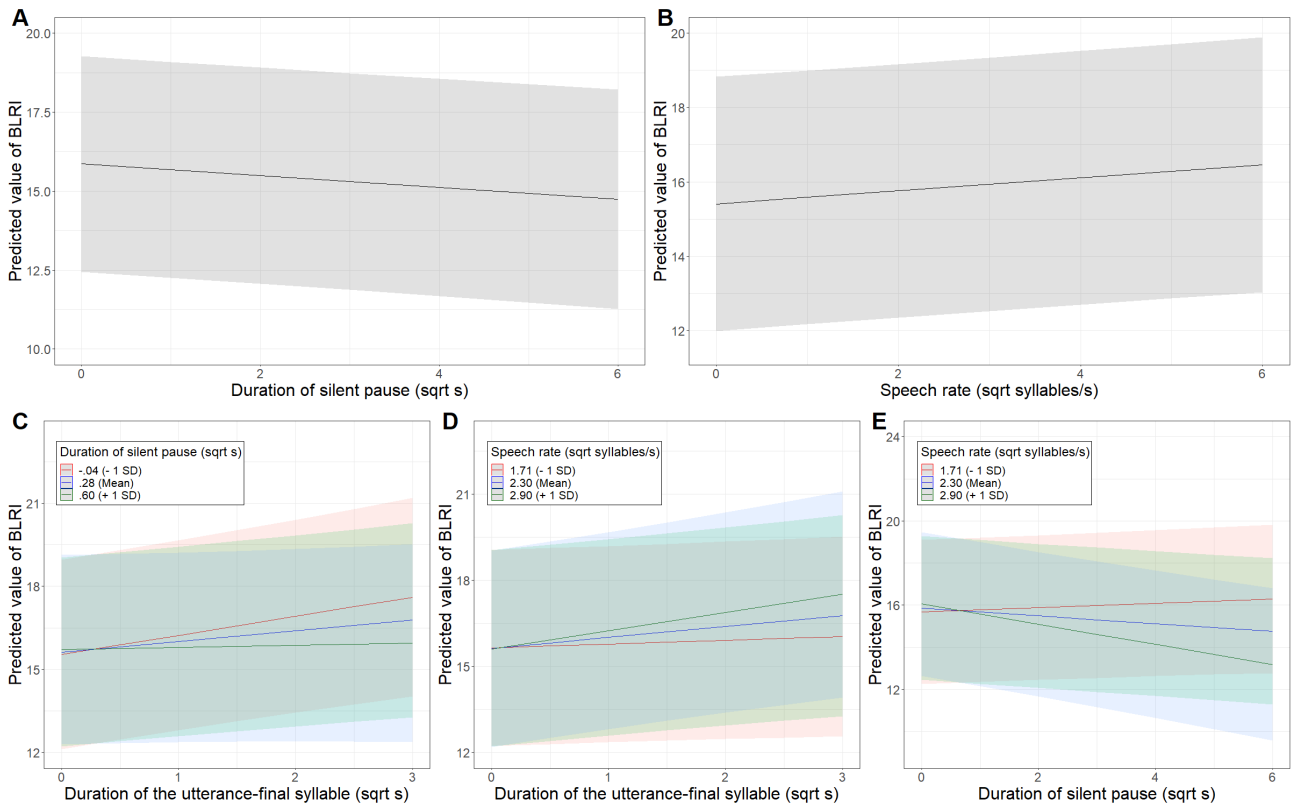


Figure 1: Results of mixed-effects regression.

line, pause duration 1 SD above mean), implying a greater magnitude of increase in the predicted value of BLRI for the same unit of increase in the duration of utterance-final syllables when the pause duration is short. Second, there are significant interaction effects between the duration of utterance-final syllables and speech rate, $\chi^2(1) = 8.101, p = .004$. Figure 1(D) shows overall positive relationships between the duration of utterance-final syllables and the predicted value of BLRI. Specifically, the slope of utterances with fast speech rate (green line, speech rate 1 SD above mean) is steeper than that of utterances with average (blue line, speech rate at mean) and slow speech rate (red line, speech rate 1 SD below mean), implying a greater magnitude of increase in the predicted value of BLRI for the same unit of increase in the duration of utterance-final syllables when the speech rate is fast. Third, the interaction effects between silent pause and speech rate are also significant, $\chi^2(1) = 21.077, p < .001$. Figure 1(E) shows that for both utterances with average (blue line, speech rate at mean) and fast speech rate (green line, speech rate 1 SD above mean), the longer the silent pause, the lower the predicted value of BLRI. Conversely, for utterances with slow speech rates (red line, speech rate 1 SD below mean), the longer the silent pause, the higher the predicted value of BLRI. There are no statistically significant three-way interaction effects, $\chi^2(1) = .078, p = .780$.

4. Discussion and Conclusions

The analyses of the main effects support our hypothesis that the duration of silent pauses is negatively associated with therapist empathy. This can be attributed to the reduced perceived willingness and affirmation and the increased hesitation of long

silent pauses [30], [31]. Next, faster speech rates of therapists is associated with a higher clients' perception of therapist empathy. This finding can be attributed to the observation that a faster speech rate also reflects a speaker's credibility, trustworthiness, and persuasiveness [35]–[38]. Our findings on therapists' speech rates add to previous studies that focused on the synchrony of speech rates between therapist and client [32]–[34].

The interaction analyses suggest that therapist empathy is generally perceived to be higher when the utterance-final syllables are longer, which can be attributed to the fact that final lengthening helps with prosodic phrasing. For instance, for utterances with short silent pauses (red line in Figure 1(C)), longer utterance-final syllables improve the predicted BLRI score by a greater magnitude than that with average (blue line) or long silent pauses (green line). This can be attributed to the stronger complementary effects of final lengthening on the prosodic phrasing of utterances with short silent pauses, and that short silent pauses *per se* may be a sign of poorer prosodic phrasing. In contrast, if the silent pause is sufficiently long (e.g., average and long silent pauses), the complementary effects brought by final lengthening may be weaker, hence leading to a smaller magnitude of improvement in therapist empathy relative to that of short silent pauses. In addition, Cantonese protracted intonation and protracted sentence-final particles can smoothen expressed emotions while emphasizing the sentential discourse functions, which may help with clients' perception of therapist empathy. In contrast, shortened intonation and particles paralinguistically intensify therapists' personal emotions, which may have negative effects on perceived

therapist empathy and hinder clients' active self-healing efforts [24]–[26].

Similar complementary effects of long utterance-final syllables can be found in their interaction effects with speech rate. For utterances with fast speech rates (green line in Figure 1(D)), longer utterance-final syllables improve the predicted BLRI score by a greater magnitude than that with average (blue line) and slow speaking rate (red line). For instance, longer utterance-final syllables can help therapists to maintain clear prosodic phrasing by final lengthening and smoothen their tone by protracted intonation and particles even when they are speaking quickly. By contrast, therapists with slow speech rates can rely on other cues (e.g., silent pauses) to maintain clear prosodic phrasing. Thus, the complementary effects on improving therapist empathy brought by long utterance-final syllables are more significant in fast speech than in slow speech.

Furthermore, our results have shown that longer pauses are associated with higher therapist empathy in utterances with slow speech rate (red line in Figure 1(E)) but not in utterances with average (blue line) and fast speech rate (green line). This can be attributed to an overall steadier prosody/rhythm in the former, while the latter two may sound more dysfluent [40]. Moreover, long pauses in utterances with fast speech rates can sound more conspicuous than that with slow speech rates. The perceived reduction in willingness and affirmation and increase in hesitation in association with long pauses may therefore be more conspicuous, leading to lower perceived therapist empathy.

This study is the first to examine the relationships between durational patterning at discourse boundaries and client rated therapist empathy in psychotherapy. Our results have shown that low-order temporal cues of prosodic phrasings, such as the duration of utterance-final syllables, silent pause, and speech rate, contribute to clients' higher-order perceptual process of therapist empathy. It is intriguing to see clients did pick up and integrate these "subtle" nuances of prosody at the discourse boundaries in their grand perceptual experience of therapist empathy. The results of the present study also have important clinical/educational implications for the use of prosody by therapists in psychotherapy, particularly on therapists' awareness and perhaps manipulation of the duration of utterance-final syllables and silent pauses when they are advised to synchronize with clients' varying speech rates [32]–[34].

5. Acknowledgments

The authors are indebted to Regine Lai and Peggy Mok for discussion and useful input. This project has been funded by the Sustainable Research Fund of The Chinese University of Hong Kong and the Early Career Scheme grant from the Hong Kong Research Grants Council (Ref.: 24604317) to which we are grateful.

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