



A model of glottal flow incorporating viscous-inviscid interaction

Tokihiko Kaburagi¹, Yosuke Tanabe²

¹Department of Acoustic Design, Faculty of Design, Kyushu University

²Graduate School of Design, Kyushu University
4-9-1 Shiobaru, Minami-ku, Fukuoka, 815-8540 Japan
kabu@design.kyushu-u.ac.jp

Abstract

A model of flow passing through the glottis is presented by employing the boundary-layer assumption. A thin boundary layer near the glottal wall influences the flow behavior in terms of the flow separation, jet formation, and pressure distribution along the channel. The integral momentum relation has been developed to analyze the boundary layer accurately, and it can be solved numerically for the given core flow velocity on the basis of the similarity of velocity profiles. On the other hand, boundary layer reduces the effective size of the channel and increases the flow velocity. Therefore, the boundary-layer problem entails viscous-inviscid interaction inherently. To investigate the process of voice production, this paper presents a method to solve the boundary-layer problem including such interaction. Experiments show that the method is useful for predicting the flow rate, pressure distribution, and other properties when the glottal configuration and subglottal pressure are specified as the phonation condition.

Index Terms: voice production, glottal flow, boundary layer, viscous-inviscid interaction

1. Introduction

To understand the mechanism of voice production, dynamic behavior of flow passing through the glottis has been studied extensively [1, 2, 3, 4]. It is known that the Reynolds number of glottal flow becomes of the order of 1000 implying that a thin boundary layer is formed near the surface of the vocal folds. It suggests that the numerical analysis of the basic equation of viscous flow (the Navier-Stokes equation) has many difficulties, since flow velocity in the boundary layer changes rapidly in the direction perpendicular to the glottal wall. In addition, the shape of the glottal channel varies during the phonatory cycle making the mesh generation problem more complicated.

On the other hand, a constitutive approach has been taken in the simulation of phonation based on the boundary-layer approximation [4]. In the method of Kármán and Pohlhausen [5], the integral momentum equation of the boundary layer can be solved numerically by assuming the similarity of velocity profiles within the layer. When the core flow velocity is given, the method can predict characteristic quantities of the boundary layer such as the thickness, wall shear stress, and the position where the boundary layer separates from the glottal wall.

The thickness of boundary layer is inversely proportional to the square root of the Reynolds number and then it can be neglected when the Reynolds number is sufficiently high. However, the Reynolds number varies during the phonatory cycle, indicating that the boundary layer can influence the effective size of the glottal channel and the core flow velocity. Therefore,

the viscous (boundary layer) and inviscid (core flow) parts of the glottal flow can interact with one another [6].

This paper presents a method to extend the ordinal one-dimensional models of the glottal flow [2, 3, 4] based on the boundary-layer assumption [5] and the viscous-inviscid interaction. Also, we use Bernoulli equation and the momentum conservation between the upstream and downstream of the separation point. By specifying the glottal configuration and the subglottal pressure, the method can estimate the volume flow rate and give numerical solution of the boundary-layer problem.

2. Boundary-layer problem with viscous-inviscid interaction

Flow through the glottis can be assumed to be incompressible and quasi-steady, since the flow velocity is sufficiently lower than the sound velocity and the typical Strouhal number is of the order of 10^{-2} [4]. A symmetrical glottal channel is modeled using the height parameter ($h(x)$) along the axis (x) and the length of the vocal folds (L_g) (Fig. 1). When the volume flow rate (U_g) is given, the nominal velocity can be expressed as

$$v_n = \frac{U_g}{h \cdot L_g} \quad (1)$$

However, development of the boundary layer reduces the effective size of the channel. If the displacement thickness is δ_1 , the effective height of the channel is $h - 2\delta_1$ and the effective (average) flow velocity is expressed as

$$v = \frac{U_g}{(h - 2\delta_1)L_g} \quad (2)$$

The effective flow velocity is greater than the nominal one, and the boundary layer may affect the separating point of the flow as well. Therefore, the boundary-layer equation should be solved to estimate the thickness of the layer and effective size of the channel.

The behavior of the boundary layer can be analyzed using the Kármán-Pohlhausen method [5] in the form of the integral momentum relation:

$$\frac{\partial}{\partial x} v^2 \delta_2 + \delta_1 v \frac{\partial}{\partial x} v = \frac{\tau}{\rho} \quad (3)$$

where δ_1 is the displacement thickness, δ_2 is the momentum thickness, τ is the wall shear stress, and ρ is the density of the fluid. Eq. (3) can be simplified by assuming the similarity of velocity profiles inside the layer, and solved numerically using, for example, the Falkner-Skan solution [5].

Using the framework of the Kármán-Pohlhausen method, Kalse et al. [6] proposed a method to solve the boundary-layer

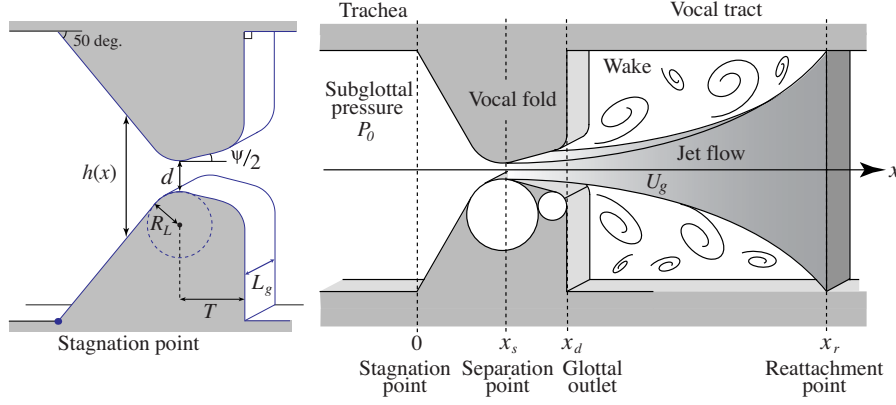


Figure 1: Model of the symmetrical glottal channel [7] controlled by the minimum height (d) and the glottal angle (ψ). L_g is the length of the vocal fold and $h(x)$ is the height of the channel. The coordinate system (x) is taken as the axis of the symmetry.

problem involving the viscous-inviscid interaction. Eq. (2) is solved jointly with the following equations

$$\frac{\delta_1^2}{\nu} \frac{dv}{dx} = f_1(H) \quad (4)$$

and

$$\nu \frac{\delta_1}{\nu} \frac{d}{dx} \left(\frac{\delta_1}{H} \right) + \left(1 + \frac{2}{H} \right) f_1(H) - H f_2(H) = 0, \quad (5)$$

where H is a function of x and called the shape factor. ν is the kinematic viscosity. f_1 and f_2 are given as

$$f_1(H) = -2.4(1 - \exp(0.43(2.59 - H))) \quad (6)$$

and

$$f_2(H) = \frac{4}{H^2} - \frac{1}{H}. \quad (7)$$

Eqs. (2)(4) and (5) constitute a set of nonlinear simultaneous equations with respect to H , δ_1 , and v . These variables are a function of x , and the problem can be solved numerically using the downstream marching along the x -axis and the Newton-Raphson method as explained below. When the values of these variables are determined as the solution of the boundary-layer problem, characteristic quantities of the boundary layer can be calculated as

$$\delta_2 = \delta_1/H \quad (8)$$

and

$$\tau = \frac{\mu v H}{\delta_1} f_2(H). \quad (9)$$

Also, the separation point of the boundary layer is estimated by finding the x -axis position where the wall shear stress becomes zero. It also indicates that $f_2(H) = 0$ and $H = 4$ at the separation point.

To solve Eqs. (2)(4) and (5) simultaneously, the differentiation with respect to x is approximated by the finite difference and the following objective functions are defined for each equation.

$$F_1 = v(x) - \frac{U_g}{(h(x) - 2\delta_1(x))L_g} \quad (10)$$

$$F_2 = \frac{\delta_1(x)^2}{\nu} \frac{v(x) - v(x - \Delta x)}{\Delta x} - f_1(H(x)) \quad (11)$$

$$F_3 = v(x) \frac{\delta_1(x)}{\nu} \left(\frac{\delta_1(x)}{H(x)} - \frac{\delta_1(x - \Delta x)}{H(x - \Delta x)} \right) \frac{1}{\Delta x} + \left(1 + \frac{2}{H(x)} \right) f_1(H(x)) - H(x) f_2(H(x)) \quad (12)$$

When the initial values are given at $x = 0$ as $H(0) = 2.55$, $\delta_1(0) = 0$, and $v(0) = U_g/(h(0) \cdot L_g)$, $H(x)$, $\delta_1(x)$, and $v(x)$ are calculated so that the values of F_1 , F_2 , and F_3 are minimized for $x = \Delta x$. According to the downstream marching, the procedure is repeated such that $x = 2\Delta x, 3\Delta x, 4\Delta x, \dots$ until the exit of the glottis.

The minimization of F_1 , F_2 , and F_3 can be performed using the Newton-Raphson method. If the variables are written as

$$\mathbf{s} = (H(x), \delta_1(x), v(x))^T, \quad (13)$$

they can be iteratively optimized such that

$$\mathbf{s}_{k+1} = \mathbf{s}_k + \Delta \mathbf{s}_k \quad (14)$$

where k is the index of the iteration. T denotes the transposition. $\Delta \mathbf{s}_k$ is the solution of linear equations

$$J \Delta \mathbf{s}_k = -F, \quad (15)$$

where $F = (F_1, F_2, F_3)^T$ and J is the Jacobian matrix for \mathbf{s}_k . The initial value \mathbf{s}_0 can be set as the optimal value of the previous position such that $\mathbf{s}_0(x) = \mathbf{s}^*(x - \Delta x)$.

3. Model of the core flow

The boundary-layer problem in the previous section can be solved when the volume flow rate (U_g) is specified as well as the configuration of the channel. To determine the value of U_g from the phonation condition of the subglottal pressure, this section describes a model of the core flow including the pressure relation based on Bernoulli's law and the momentum conservation between the upstream and downstream of the separation point.

3.1. Pressure distribution along the channel

The origin of the x -axis is taken at the stagnation point of the glottal inlet. Also, x_s , x_d , and x_r respectively represent the point of separation, glottal outlet, and point of reattachment as shown in Fig. 1. When the subglottal pressure is given as P_0 , the pressure distribution $P(x)$ along the glottal channel can be expressed using the Bernoulli pressure P_B and the viscous friction loss P_v [4]. Bernoulli's law for the incompressible flow can be written as

$$\rho \frac{\partial \varphi}{\partial t} \Big|_{x=0} + \frac{1}{2} \rho v^2 \Big|_{x=0} + P_0 = \rho \frac{\partial \varphi}{\partial t} + \frac{1}{2} \rho v^2 + P_B, \quad (16)$$

where φ is the velocity potential, v is the effective flow velocity, and ρ is the air density. The left and right sides of Eq. (16) respectively correspond to the Bernoulli function at the origin and at a point along the x axis. The velocity v can be expressed using the volume flow rate (U_g), length of the vocal fold (L_g), and effective height of the channel (h) as

$$v = \frac{U_g}{A} = \frac{U_g}{L_g \cdot h}, \quad (17)$$

where A is the effective sectional area. The second term of Eq. (16) can be neglected since $h(x) \ll h(0)$. Also, by expressing the velocity potential as

$$\varphi(x) = \int_{-\infty}^x v(\tau) d\tau, \quad (18)$$

Eq. (16) can be rewritten as

$$P_B = P_0 - \frac{1}{2}\rho \left(\frac{U_g}{L_g h(x)} \right)^2 - \frac{\rho}{L_g} \int_0^x \frac{1}{h(\tau)} d\tau \frac{dU_g}{dt}. \quad (19)$$

P_B is then obtained from the values of P_0 and U_g .

The friction loss P_v can be expressed based on Poiseuille's flow as

$$P_v = \frac{12\mu}{L_g} U_g \int_0^x \frac{1}{h(\tau)^3} d\tau, \quad (20)$$

where μ is the viscous coefficient. Finally, the pressure distribution is obtained by subtracting P_v from Eq. (19) as

$$P(x) = P_0 - \frac{1}{2}\rho \left(\frac{U_g}{L_g h(x)} \right)^2 - \frac{\rho}{L_g} \int_0^x \frac{1}{h(\tau)} d\tau \frac{dU_g}{dt} - \frac{12\mu}{L_g} U_g \int_0^x \frac{1}{h(\tau)^3} d\tau. \quad (21)$$

3.2. Estimation of the volume flow rate

To estimate the volume flow rate U_g , the following assumptions are made. First, the energy loss in the glottal jet is ignored for the interval between the separation point and the glottal outlet [4]. Then the pressure at the separation point is maintained as $P(x) = P(x_s)$ for $x_s \leq x \leq x_d$. Second, the sectional area of the glottal jet is constant so that $A(x) = A(x_s)$ for $x_s \leq x \leq x_d$. Finally, the pressure at the reattachment point is equal to the atmospheric pressure and normalized as $P(x_r) = 0$.

The momentum preservation between the upstream and downstream of the separation point suggests the following relation

$$P(x_d)A(x_r) + \frac{\rho U_g^2}{A(x_d)} = P(x_r)A(x_r) + \frac{\rho U_g^2}{A(x_r)}. \quad (22)$$

Also, from the above assumptions, it is implied that $P(x_d) = P(x_s)$, $A(x_d) = A(x_s)$, and $P(x_r) = 0$. Then it follows that

$$P(x_s)A(x_r) + \frac{\rho U_g^2}{A(x_s)} = \frac{\rho U_g^2}{A(x_r)}. \quad (23)$$

Next, the time differentiation in Eq. (21) is approximated as

$$\frac{dU_g}{dt} \doteq \frac{U_g - \tilde{U}_g}{\Delta t}, \quad (24)$$

where Δt is the time difference and \tilde{U}_g is the flow rate of the previous time step. If the sectional area at the separation point is given as $A(x_s) = L_g \cdot h(x_s)$, Eq. (21) can be written as

$$P(x_s) = P_0 - \frac{1}{2}\rho \left(\frac{U_g}{A(x_s)} \right)^2 - a(U_g - \tilde{U}_g) - bU_g. \quad (25)$$

From Eqs. (23) (25), it is derived that

$$MU_g^2 + (a+b)U_g - (P_0 + a\tilde{U}_g) = 0 \quad (26)$$

and then U_g is given as

$$U_g = \frac{-(a+b) + \sqrt{(a+b)^2 + 4M(P_0 + a\tilde{U}_g)}}{2M}. \quad (27)$$

Parameters in Eq. (27) are given as

$$a = \frac{\rho}{L_g \Delta t} \int_0^{x_s} \frac{1}{h(x)} dx \quad (28)$$

$$b = \frac{12\mu}{L_g} \int_0^{x_s} \frac{1}{h(x)^3} dx \quad (29)$$

$$M = \frac{\rho}{2A(x_s)^2} (1 - 2N + 2N^2) \quad (30)$$

and

$$N = \frac{A(x_s)}{A(x_r)}. \quad (31)$$

Eq. (27) indicates that U_g is determined by the sectional area $A(x_s)$ as well as the channel height h and subglottal pressure P_0 . When the flow is quasi-steady, the unsteady term of Eq. (16) can be neglected and the parameter a can be set to zero.

4. Numerical results

When the glottal configuration and the subglottal pressure are specified, the estimation of the volume flow rate and the solution of the boundary-layer problem are performed as follows. First, the separation point (x_s) is set at the glottal position where the channel height is at a minimum. U_g is then estimated using the relation in Eq. (27), where the parameters are set to $a = 0$ and $b = 0$ for simplicity. Next, the boundary-layer problem is solved for the initial value of U_g , and the separation point is estimated. U_g is calculated again using the estimated separation point, where the effective height of the glottal channel is obtained by subtracting the thickness of the boundary layer. Finally, the boundary-layer problem is solved again with the new value of U_g . The pressure distribution along the channel is also calculated from Bernoulli's relation.

The shape of the vocal fold is represented using the model proposed by Scherer *et al.* [7]. The model can be controlled by parameters representing the minimum glottal height (d) and the tilt angle of the vocal fold (ψ) as shown in Fig. 1. Other parameters of the model is set as $T = 0.3$ cm and $L_g = 1.2$ cm. The height of the trachea and vocal tract is set at 1.73 cm. The air density is $\rho = 1.184 \times 10^{-3}$ g/cm³ and the viscous coefficient is $\mu = 1.82 \times 10^{-4}$ g/cm·s.

In Fig. 2, the glottal angle is changed as $\psi = 0, 4, 8, 12$ deg while the minimum height and the subglottal pressure are set at $d = 0.04$ cm and $P_0 = 8$ cmH₂O, respectively. The solid, broken, and thick lines respectively represent the glottal shape, displacement thickness of the boundary layer, and the pressure distribution. The circle plotted on each vocal fold shows the estimated point of flow separation. The estimated volume flow rate and the Reynolds number are also shown in the figure. On the other hand, Fig. 3 shows the result for the change of the minimum glottal height, where d is set at 0.01, 0.02, 0.04, and 0.08 cm, respectively. The glottal angle is $\psi = 8$ deg and the subglottal pressure is $P_0 = 8$ cmH₂O. These results indicate that the separation point and the pressure distribution are both affected by the glottal configuration. When the glottal angle

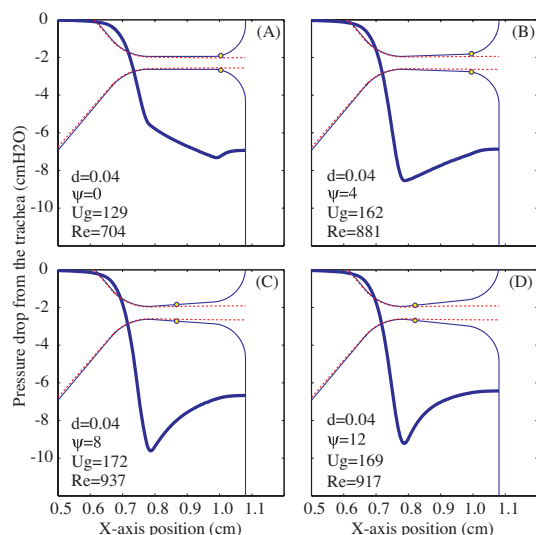


Figure 2: Boundary-layer thickness, separation point, and pressure distribution in the change of the glottal angle.

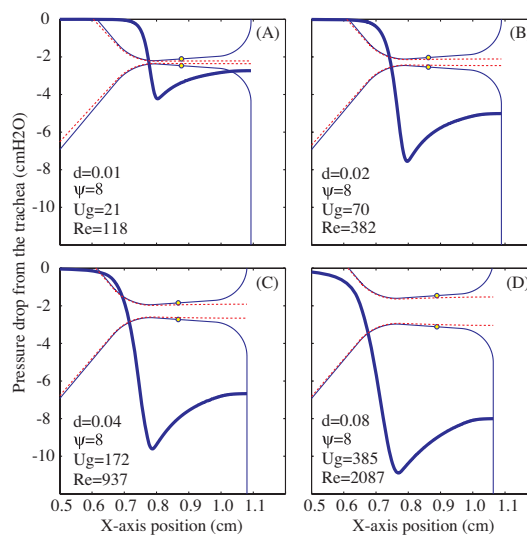


Figure 3: Boundary-layer thickness, separation point, and pressure distribution in the change of the minimum glottal height.

is positive ($\psi > 0$), the channel takes a divergent configuration. Figure 2 shows clearly that the separation point moves to the upstream direction by increasing the glottal angle. The flow rate (U_g) and the Reynolds number (Re) shown in Fig. 3 are changed significantly, and the separation point in (D) is also shifted slightly to a downstream position. Finally, predicted pressure distribution is compared with that obtained by a flow measurement [7] in Fig. 4, showing that a good agreement can be obtained by the proposed flow analysis method.

5. Conclusions

A model of glottal flow was presented by incorporating the interaction of the core flow and the boundary layer. A number of numerical results were also shown, indicating that the model is capable of predicting basic properties of the flow adequately. Further study will be performed for the dynamic simulation of phonation by combining a mechanical model of the vocal fold with the proposed flow model. It is noteworthy that our flow model can predict the wall shear stress conveniently as well as the pressure to investigate its effect on the movements of the vocal folds. This research was partly supported by the Grant-in-Aid for Scientific Research from the JSPS (Grant No. 18500134 and 19103003).

6. References

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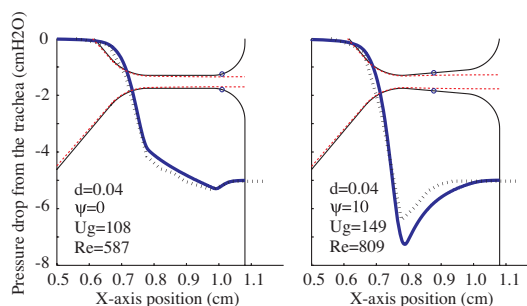


Figure 4: Comparison of the predicted (solid lines) and measured [7] (broken lines) pressure distributions. The minimum height is 0.04 cm and the glottal angle is 0 (left) and 10 deg (right). The volume flow rate was determined so that the transglottal pressure difference was set to 5 cmH₂O.

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