

ROBUST VECTOR QUANTIZATION FOR CHANNELS WITH MEMORY

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ABSTRACT

This study focuses on two issues: parametric modeling of the channel and index assignment of codevectors, to design a vector quantizer that achieves high robustness against channel errors. We first formulated the design of a robust zero-redundancy vector quantizer as a combinatorial optimization problem leading to a genetic search for the minimum distortion index assignment. This study also presents an index assignment algorithm based on Gilbert's model with parameter values estimated using a real-coded genetic algorithm. Simulation results indicate that the global explorative properties of genetic algorithms make them very effective at estimating Gilbert's model parameters and by using this model the index assignment can be developed to respond to channel conditions.

Keywords: channel modeling, index assignment

1. INTRODUCTION

Vector quantization (VQ) has been widely applied in speech and image coding for data compression [1]. Transmitting vector-quantized data over a noisy channel changes the encoded information and consequently leads to severe distortions in the reconstructed output. The channel distortion can be greatly reduced by assigning suitable indices to the codevectors of non-redundant VQ coding system [2,3]. Finding the best index assignment requires searching every possible codebook permutation for the one that yields the minimum distortion under noisy channel conditions. However, because the nature of this solution is NP-complete, it requires enormous computational complexity which, for even small size codebooks, may be prohibitive. Previous research on index assignment concentrates on mathematically simple memoryless binary symmetric channels. Unfortunately, however, transmission errors encountered in digital communication channels exhibits various degrees of statistical dependencies that are contingent on the transmission medium and on the particular modulation and demodulation technique.

Further improvement can only be realized through a more precise characterization of the channel on which the design of the index assignment is based. Many parameterized probabilistic models have been proposed to characterize channels with memory. The best known

of these models is Gilbert's two-state Markov chain model [4]. The model is popular because it is relatively simple and can characterize a large variety of digital channels. Traditionally the optimal identification of Gilbert's model has been dealt with using the gradient-descent algorithm [5]. The basic problem with this approach is that its simple downhill search transitions can easily trap into local minima. Using the genetic algorithm, we attempt to capitalize more fully on the properties of the channel and then develop a channel-matched index assignment for use in robust vector quantization.

2. GENETIC ALGORITHMS

Genetic algorithms (GAs) are powerful stochastic search techniques based on the laws of natural selection and genetics. The main attraction of GAs arises from the fact that the given search space is explored in parallel by means of iterative modifications of a population of individuals. Each individual in the population, called a chromosome, represents a potential solution to a given problem. Choosing an appropriate representation of chromosomes is the first step in applying GAs to solve any optimization problem, which conditions all subsequent steps of the implementation. In our problem domain, we employ two encoding schemes to formulate the genetic structure of chromosomes used in GA optimization process. Specifically, the study applies real-coded GA to the Markov characterization of error sequences, and the GA with permutation representation to the index assignment problem.

GAs start with an initial population in which the chromosomes are randomly generated. To provide successively better solutions to the problem, these chromosomes will undergo the probabilistic selective process, crossover operation and mutation operation through multiple generations. In the selective process, the number of offspring allocated to each parent chromosome is determined by its evaluated performance. To prevent premature convergence of the population, we employ an adaptive selection mechanism [6] that sorts individuals by descending order of fitness and then assigns the expected number of offspring according to their relative ranking. The crossover operator produces offspring by recombining the genetic material of fitter chromosomes. This allows offspring to inherit the beneficial genes from each parent and thereby be superior to either parent. After that, with a small probability, the mutation operator is

used to introduce random variations into the genetic structure of the chromosomes. The mutation operator aims to prevent the premature loss of diversity in the population with the search converging to a local optimum. When the maximum number of generations has been reached, the best chromosome in the final population is taken as the GA's solution for functional optimization.

3. MARKOV CHARACTERIZATION OF ERROR SEQUENCES

The vector quantizer operates by employing a multipath search that pursues a set of alternatives and chooses among them the best possible output sequence. But when transmitting VQ data over noisy channels, channel errors are often introduced in the bits that convey information about codevector indices. To achieve high robustness against channel errors, we formulate the channel robust VQ design as a combinatorial optimization problem that leads to a search for the minimum distortion index assignment. Past investigations of discrete memoryless channels provide insufficient information regarding the transmission of VQ data over channels with memory. Further improvement can only come through intelligent exploitation of the statistical dependencies between error occurrences.

To design a more sophisticated error protection scheme requires that parameterized probabilistic models be used to summarize some of most relevant aspects of error statistics. Gilbert's two state Markov chain model [4], although simple, is accurate enough to enable a detailed investigation into the effect of channel errors on vector quantizer. The model state transition diagram is illustrated in Fig. 1. In the bad state q_1 errors occur with the probability $1-h$; while in the good state q_0 errors do not occur. The state transition probabilities are P and p for the q_0 to q_1 and q_1 to q_0 transitions, respectively. The principal difficulty using Gilbert's channel characterization is that the parameters $\{P, h, p\}$ are not directly observable, so methods of deducing the parameters from easily measured error-gap distribution are considered here. This distribution, denote $P(0^k|1)$, can be well approximated by the sum of two exponential functions:

$$G(k) = \alpha_1 \beta_1^k + \alpha_2 \beta_2^k \quad (1)$$

Proceeding in this way, the original descriptive modeling issue can be formulated as one of the constrained nonlinear optimization problems that can be stated as follows:

$$\min_{\alpha, \beta} E_g = \min_{\alpha, \beta} \left\{ \sum_{k=1}^K \frac{1}{k} [\log_{10} P(0^k | 1) - \log_{10} G(k)]^2 \right\} \quad (2)$$

subject to the following constraints

- (i) $\alpha_1 + \alpha_2 = 1$ and
- (ii) $0 \leq \alpha_i \leq 1, 0 \leq \beta_i \leq 1, \text{ for } i = 1, 2,$

where $K=2000$ is empirically determined to be the longest interval between two consecutive errors. Given

that the optimal values of $\{\alpha_1, \beta_1, \alpha_2, \beta_2\}$ have been determined, Gilbert's model parameters $\{P, h, p\}$ can be calculated as follows [4]:

$$h = \frac{\beta_1 \beta_2}{\beta_1 - \alpha_1 (\beta_1 - \beta_2)}, \quad (3)$$

$$P = \frac{(1 - \beta_1)(1 - \beta_2)}{1 - h}, \quad (4)$$

$$p = \alpha_1 (\beta_1 - \beta_2) + \frac{(1 - \beta_1)(\beta_2 - h)}{1 - h}. \quad (5)$$

Traditionally, the gradient-decent algorithm [5] has been used to identify the optimal Gilbert's model parameters, for which the objective function E_g is minimal. While this approach converges rapidly, it suffers from the problem of convergence toward local minima that are critically dependent upon the starting point. Recognizing this, we employ the genetic algorithm which performs a multiple directional search by maintaining a population of potential solutions. The solution is defined by two pairs of parameters $\{\alpha_i, \beta_i\}$ and hence, can be encoded into a chromosome as a list of real numbers, that is, $\mathbf{S} = \{s_1, s_2, s_3, s_4\} = \{\alpha_1, \beta_1, \alpha_2, \beta_2\}$. The fitness values of all chromosomes were ranked with respect to the objective function $F(\mathbf{S}) = 1/E_g$ using equation (2). A two-point crossover is performed by choosing two crossing points in the strings of two chromosomes and by exchanging the substrings between these two points. Initiated by the merits of simulated annealing (SA), we propose a new mutation operator that uses the probabilistic acceptance test technique internally to determine whether to accept the mutated solution, or just stay with the previous solution. This search will move towards some feasibility regions likely to contain the global optimum, but escape from local optima is still possible since inferior solutions are allowed. The parameter values used for the maximum number of generations, the population size, the crossover rate, and the mutation rate were empirically determined to be 1000, 50, 0.6, and 0.1, respectively.

4. MINIMUM DISTORTION INDEX ASSIGNMENT

For transmission of VQ data over a noisy channel, the distortions due to channel impairments can be greatly reduced by assigning suitable indices to elements of the codebook. For a fixed codebook and a given channel, every possible index assignment γ should be searched for the one that minimizes the channel distortion $D_c(\gamma)$ expressed as follows:

$$D_c(\gamma) = \sum_{i=1}^M \sum_{j=1}^M P(\mathbf{c}_i) P(\gamma(\mathbf{c}_j) | \gamma(\mathbf{c}_i)) d(\mathbf{c}_i, \mathbf{c}_j) \quad (6)$$

where $\gamma(\cdot)$ is the index assignment function and $C = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M\}$ represents the codebook. The effectiveness of the optimization algorithm crucially depends on how the channel transition probabilities $P(\gamma(\mathbf{c}_j) | \gamma(\mathbf{c}_i))$ are specified in advance. Gilbert's error source model has two states that constitute a hidden Markov model (HMM).

Referring to Fig. 1, the sequence of errors e_1, e_2, \dots, e_m is generated by the HMM with initial state distribution

$$\pi = (\pi_0, \pi_1) = \left(\frac{p}{P+p}, \frac{P}{P+p} \right) \quad (7)$$

and matrices $\{\mathbf{A}, \mathbf{B}\}$ of the form

$$\mathbf{A} = \begin{pmatrix} 1-P & P \\ p & 1-p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ h & 1-h \end{pmatrix} \quad (8)$$

where \mathbf{A} is the matrix of transition probabilities and \mathbf{B} represents the observation probability distribution. Following Turin [7], we describe the channel transition probability on the Gilbert's channel using matrix algebra as follows

$$P(\gamma(\mathbf{c}_j) | \gamma(\mathbf{c}_i)) = P(e_1, e_2, \dots, e_m) = \pi \prod_{k=1}^m \mathbf{P}_e(e_k) \mathbf{1} \quad (9)$$

where $\mathbf{1}$ is a vector of ones and the matrix $\mathbf{P}_e(e_k)$ in the form of

$$\mathbf{P}_e(0) = \begin{pmatrix} 1-P & Ph \\ p & (1-p)h \end{pmatrix}, \quad \mathbf{P}_e(1) = \begin{pmatrix} 0 & P(1-h) \\ 0 & (1-p)(1-h) \end{pmatrix} \quad (10)$$

Since there are $M!$ possible combinations of indices to the codebook of size M , the index assignment problem belongs to the class of NP-hard problem. In this work, we propose an index assignment scheme based on the genetic algorithm that proves very effective in controlling the performance degradation caused by channel error. The input to the proposed algorithm consists of a domain-specified codebook and Gilbert's model parameters of a given channel that will carry the codevector indices. The fitness value for each chromosome is $-D_c(\gamma)$. Since the search space of interest is a set of permutations of the indices, we can encode each chromosome as a list of indices that number the representing codevectors. For example, a chromosome to a codebook of size 4 may be $\mathbf{S}=[3,2,1,4]$, where the number n_j in the position j of the list gives the new position to which the j -th codevector would be located. For GAs with permutation representation, the classical crossover operators usually yield illegal offspring in that some indices may be missed while some indices may be duplicated. To compensate for this shortage, we employ a partial-mapped crossover operator [8] that incorporates a repairing procedure as an add-on extra to the simple two-point crossover. The mutation considered here is to invert the substring of an existing chromosome between two randomly selected positions.

5. EXPERIMENTAL RESULTS

To test the validity of various channel modeling algorithms, a series of sample error sequences are generated by using a digital channel simulator [7]. These error sequences are 100,000 bits long. The channel condition is defined by the Gilbert's error source model whose parameters have been determined from the statistical measurements of a real telephone channel [4]: $\{P, p, h\} = \{0.003, 0.034, 0.84\}$. Table I compares the gradient and genetic algorithm approaches for Markov characterization of error sequences. The results

demonstrate that the genetic algorithm has the characteristics of a global search capability and is therefore much better than the gradient method. To elaborate further, Fig. 2 shows the error-gap distribution and its Gilbert's modeled fit. The good agreement between them provides the justification for asserting the GA's ability to estimate Gilbert's model parameters and demonstrates its usefulness in characterizing noisy channels with memory.

The next step of investigation is concerned with the potential advantages of the genetic algorithm in improving the robustness of non-redundant VQ coding systems. The input signals considered here include first order Gauss-Markov sources described by $x(n) = \rho x(n-1) + w(n)$, where $w(n)$ is zero-mean, unit-variance white Gaussian noise, with correlation coefficient $\rho = 0.5$. This was tested with rate $R = 1$ bit/sample vector quantizers having the following codebook size and dimension value $(M, N) : (16, 4), (64, 6), (256, 8)$. The codebooks were designed for a noiseless channel using the standard generalized Lloyd algorithm. Table II presents the results of the vector quantizer in conjunction with different index assignment algorithms for the case where the bits in the codevector indices are subjected to error sequences typical of the Gilbert's channel. The results clearly demonstrate the improved performance achievable using binary search algorithm (BSA) and genetic algorithm (GA) in comparison to the performance of random index assignment. A comparison between BSA and GA also suggested that the genetic search strategy converges to a point where the corresponding average distortion is more consistent from run to run. Fig. 3 compares the overall signal-to-noise ratio (SNR) of the vector quantizer for the Gilbert's channel. Simulation results indicate that the accuracy of the channel model used in developing the index assignment algorithm is extremely important to the performance of the vector quantizer.

6. CONCLUSION

This study presents a novel means of exploiting Markov characterization of error sequences in the design of a robust vector quantizer for channels with memory. We first emphasized the importance of matching the real channel behavior to the channel model on which the index assignment design is based. This task was accomplished by using Gilbert's two-state Markov chain model to characterize the statistical dependencies in relative occurrences of errors. Simulation results indicate that the real-coded genetic algorithm coupled with an annealing mutation operator leads to greater accuracy for the estimation of Gilbert's model parameters. Finally, the merits of using the genetic algorithm for assigning binary indices to the VQ codevectors are explored. It is concluded that with the aid of Gilbert's channel characterization the index assignment algorithm can be developed to better track the intrinsic nature of channel errors.

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REFERENCES

- [1] R. M. Gray, "Vector quantization," *IEEE Acoust. Speech Signal Processing Mag.*, vol. 1, pp. 4-29, Apr. 1984.
- [2] K. Zeger and A. Gersho, "Pseudo-Gray coding," *IEEE Trans. Commun.*, Vol. 38, No. 12, pp. 2147-2158, Dec. 1990.
- [3] Farvardin, "A study of vector quantization for noisy channels," *IEEE Trans. Inform. Theory*, vol. 36, pp. 799-809, July 1990.
- [4] E. N. Gilbert, "Capacity of a burst-noise channel," *Bell Syst. Tech. J.*, vol. 39, pp. 1253-1265, Sept. 1960.
- [5] J. Y. Chouinard, M. Lecours and G. Y. Delisle, "Estimation of Gilbert's and Fritchman's models parameters using the gradient method for digital mobil radio channels," *IEEE Trans. Veh. Technol.*, vol. 37, no. 3, pp. 158-166, August. 1988.
- [6] D.E. Goldberg, *Genetic Algorithm in Search, Optimization and Machine Learning*, New York: Addison-Wesley, 1989.
- [7] W. Turin, *Performance Analysis of Digital Transmission Systems*. New York: Computer Science Press, 1990.
- [8] D. Goldberg and R. Lingle, "Alleles, loci and the traveling salesman problem", in *Proc. First Int. Conf. on Genetic Algorithms*, J. Grefenstette, editor, Lawrence Erlbaum Associates, Hillsdale, NJ, 1985.

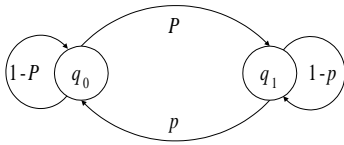


Fig. 1. Gilbert's channel model.

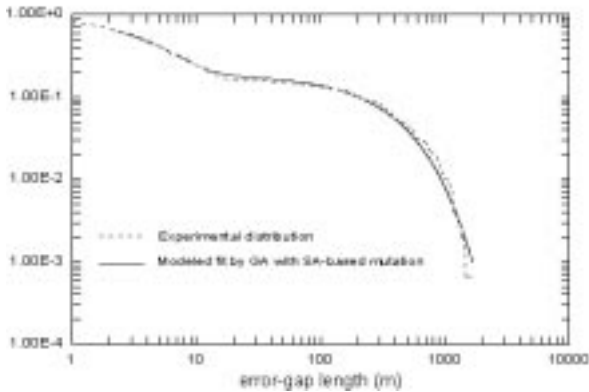


Fig. 2. Experimental err-gap distribution and its Gilbert model fit.

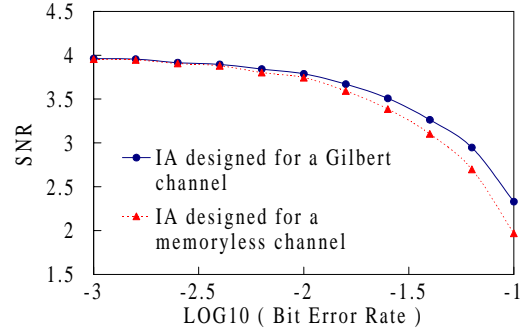


Fig. 3. SNR (dB) performance of a vector quantizer with index assignment (IA) on a Gilbert channel.

Table. I. MSE distortions of gradient and genetic algorithm approaches in five different runs.

	Gradient	Genetic Algorithm
Run #1	0.080675	0.028442
Run #2	0.166645	0.028442
Run #3	0.156661	0.028442
Run #4	0.112632	0.028442
Run #5	0.032546	0.028442
Average	0.109832	0.028442

Table. II. Channel MSE for different vector quantizers in five different runs of the binary switching algorithm, genetic algorithm and for the ensemble average of $M!$ codes.

	N=4		N=6	
	BSA	GA	BSA	GA
Run #1	0.1803	0.1802	0.3174	0.2965
Run #2	0.1856	0.1802	0.3091	0.2967
Run #3	0.1806	0.1802	0.3132	0.3009
Run #4	0.1810	0.1802	0.3104	0.2983
Run #5	0.1811	0.1802	0.3176	0.2968
Average	0.1817	0.1802	0.3136	0.2979
Random	0.2811		0.5361	