



## NOISE QUALITY IMPROVEMENT THROUGH SVD EQUALIZATION

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### ABSTRACT

In the modern computer environments, special software is available for noise generation. Depending on the quality of the software, usually also related to the hardware, the spectral flatness of the obtained noise is generally acceptable for long sequences. For shorter sequences the open loop procedure usually utilized by the previously mentioned software fails to give the expected results. For number of samples less than one thousand the spectral flatness which is equivalent to the eigen value spread needs to be improved. A new method is proposed here based on the SVD analysis technique which leads to the improvement of the spectral flatness. For a given noise frame an algorithm is provided giving the possibility to set the eigenvalues to equal magnitudes which has as consequence the regeneration of a noise with increased spectral flatness. Simulation results gave significant improvement for both uniform and gaussian noise distributions.

### 1. INTRODUCTION

Noise generation or random number generation is very important and useful process in many different kinds of applications [1] as simulation, cryptography, signal processing, communications, control theory etc to name but a few. Moreover it is a standard utility of any modern computer environment.

a) Simulation: Many phenomena are random in nature so when a computer is being used to simulate these phenomena random numbers are required to approach reality. Simulation covers a great number of fields extending from the study of nuclear phenomena, where particles are subject to random collisions, to operational research.

b) Cryptography: The major goal here is to encode a message with a special manner such that it cannot be

deciphered by anyone but the intended recipient. One way to accomplish this is to make the message look random by using random number sequences to encode the message in a proper way so that the recipient can use the same random sequence to decode it, only if he possesses the adequate key [2].

c) Signal Processing: The use of random number generators is very important in signal processing. Dithering techniques are well known tools used for the improvement of the quality of audio and other signals. Unvoiced sounds in speech processing are usually modeled as the output of a linear all pole system excited by a random sequence.

d) Communications: Random number generators are important for the creation and testing of queuing models. Data scramblers, on the other hand are usually used as a basic component of a modem.

e) Control Theory: In system identification it is important to excite an unknown system with a signal having a flat spectrum in order to assure a response which best describes the system.

f) Modern Computer Environment. Many computer algorithms are frequently tested in the presence of noise for examining their robustness and effectiveness. Noisy data has a small eigenvalue spread and corresponds to the best arithmetic condition for a given algorithm as far as the data is concerned.

Long enough sequences (several thousand samples) produced by a good quality noise generator have almost flat spectrum which is the characteristic of white noise.

Dealing with much shorter random sequences, we observe that spectral fluctuations occur and become more severe as the length of the sequences decreases. This simply indicates that locally the noise generator fails to produce flat spectrum, so a proper process would be desirable which when applied to the noise sequence it will result another sequence with more flat spectrum.

A new method is proposed here based on the Singular Value Decomposition (SVD) technique which clearly leads

to the improvement of the spectral flatness of a given noise sequence, as it will be shown subsequently. The paper is organized as follows: Section 2 deals with the decomposition of a signal using singular values while in section 3 the equalization idea will be applied to the decomposition of a noise sequence in order to improve the noise quality. In section 4 experimental results are presented which show the improvements obtained.

## 2. SIGNAL DECOMPOSITION USING SINGULAR VALUES

Let us consider a real data record of  $N$  signal samples  $\{x(0), x(1), \dots, x(N-1)\}$  and construct a corresponding  $(N-L+1) \times L$  observation matrix  $X$  with Toeplitz structure:

$$X = \begin{bmatrix} x(L-1) & x(L-2) & \dots & x(0) \\ x(L) & x(L-1) & \dots & x(1) \\ \dots & \dots & \dots & \dots \\ x(N-1) & x(N-2) & \dots & x(N-L) \end{bmatrix} \quad (1)$$

where  $L$  is defined as signal order. Its definition is necessary to set up the observation matrix [2][3][4]. In the case of predictable signals this order is equal to the memory of the predictor. Generally  $L$  is related to the acceptance of a shift invariance property valid for segments of the signal of length  $L$ . It is known that such an observation matrix may be decomposed into a sum of rank one matrices as follows:

$$X = \sum_{k=1}^L \sigma_k u_k v_k^T \quad (2)$$

where  $\sigma_k$  are the singular values of the matrix  $X$  and  $u_k$  and  $v_k$  are the eigen vectors of matrices  $X^T X$  and  $XX^T$  respectively. This decomposition corresponds to an optimization procedure in the following sense:

The eigenvectors associated to the smallest singular values convey information related to the signal structure in terms of exponential sinusoids contained in the signal. In the case of a noise sequence the previous decomposition has the trend to include more or less equal singular values and random eigenvectors. Relationship (2) will be used next as a basic generation tool in order to produce signal samples with improved characteristics.

## 3. DECOMPOSING A NOISE - THE EQUALIZATION IDEA

For short data segments the singular values computed through SVD are less and less equal. The method proposed here consists of setting the singular values equal to each other first, an assumption valid for ideal noise. The whole method will be organized as follows:

a) The short noisy frame is analyzed using the SVD technique. A set of singular values is obtained together with a set of eigenvectors.

b) The mean of the obtained eigenvalues is computed:

$$\sigma = \frac{1}{L} \sum_{i=1}^L \sigma_i \quad (3)$$

c) The following relationship is used in order to generate again matrix  $X'$ :

$$X' = \sigma \sum_{i=1}^L u_i v_i^T \quad (4)$$

d) As  $X'$  is no more a Toeplitz matrix, this step consists of computing a new  $X''$  matrix which has the Toeplitz structure and corresponds thus to a new noise sequence. A method is proposed in [5] and it is shown [6] that it corresponds to a special filtering procedure. It consists of computing the average of the diagonal elements of matrix  $X'$  and use them to form the matrix  $X''$  which will be Toeplitz. The new noise sequence will be the vector generating this Toeplitz matrix.

The previous procedure is used iteratively. The new noise will be analyzed again by the SVD method and the steps (a)-(d) will be reexecuted. During the second iteration the singular values obtained are generally different from the initial ones and tend to be more equal each one to the other. It is easy, using a thresholding operation, to find a criterion which will stop the iterative procedure. Generally speaking, the whole process converges to an improved noise which results as a better trade-off between the degradation after the filtering corresponding to the forced Toeplitz structure of the matrix  $X'$  and the equalization operation. Note that the equalization strategy used previously is fully justified by the fact of the equal distribution of the energy into the singular values of a truly noise sequence.

#### 4. EXPERIMENTAL RESULTS AND IMPROVEMENT OF THE NOISE QUALITY FOR SHORT NOISE RECORDS

Many experiments were conducted in order to show the effectiveness of the previously described method. The initial noise sequences were obtained from a random number generator with uniform distribution. The noise quality is assessed by the flatness or equivalently the variance of its spectrum. The plot of the spectrum variance before (solid line) and after equalization with only one iteration (dashed line) versus the number of the singular values used in the analysis procedure is shown in Figure 1. The curves result from the average of 100 trials with different noise seed each and a noise length of 60 samples. It can be observed that the spectrum variance of the noise after equalization decreases as the number of the singular values used

increases. It can be seen that for 25 singular values the new noise spectrum is twice as flat as that before equalization.

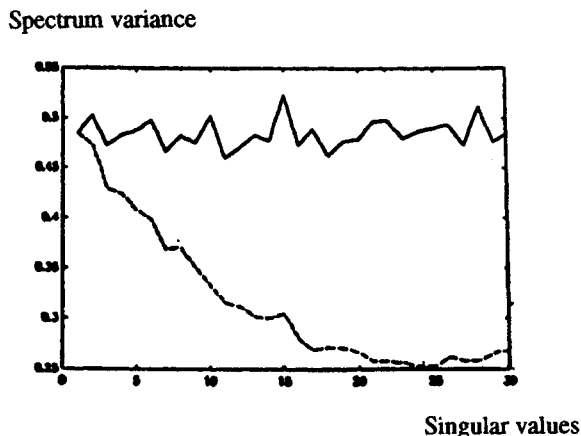


Figure 1. Plot of the spectrum variances of 60 noise samples (solid line) and the variances after SVD equalization versus the number of singular values.

In Figure 2 the plots of the normalized average spectrum variances resulting from ten noise records before (solid line) and after equalization (dashed line) versus the number of iterations are shown. It is obvious that the iterative procedure lessens even more the spectrum variance and converges after a small number of iterations. For

this experiment the record length was 100 samples and 30 singular values were used.

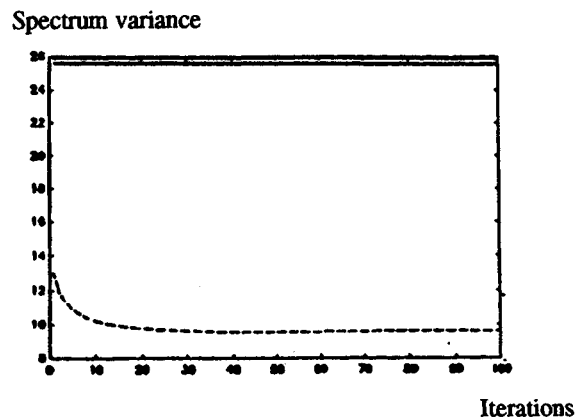


Figure 2. Spectrum variance (normalized) before (solid line) and after (dashed line) SVD equalization.

In figures 3 and 4 the plots of the ratios of the spectrum variances after SVD equalization to the spectrum variances of the initial noise with uniform and gaussian distributions respectively are given. In the abscissa the trial number is shown. The mean of spectrum variance improvement for all these trials is 1.98 for the uniform distribution and 5.63 for the gaussian distribution.

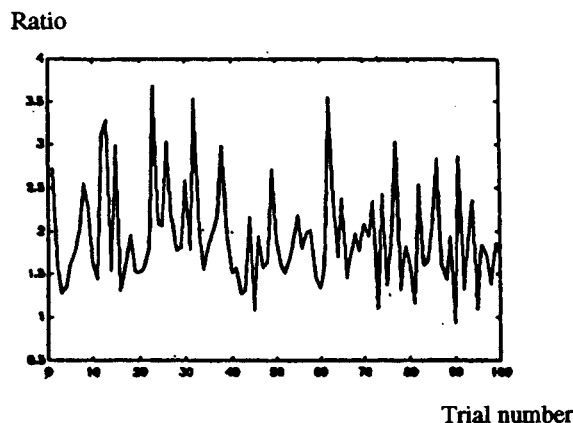


Figure 3. Plot of the ratio of the spectrum variance after SVD equalization to the spectrum variance of the initial noise of uniform distribution.

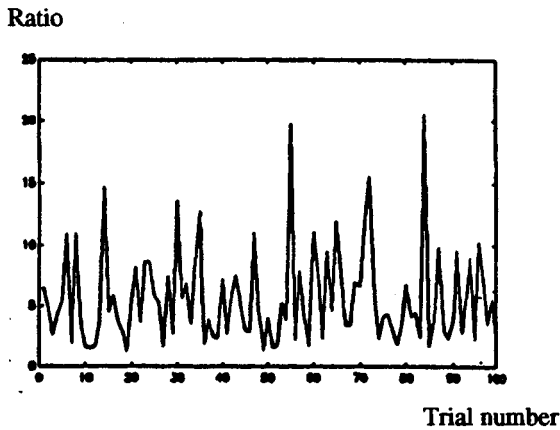


Figure 4. Plot of the ratio of the spectrum variance after SVD equalization to the spectrum variance of the initial noise of gaussian distribution.

Finally, in Figure 5 it is shown that the SVD equalization does not alter the noise distribution. Figure 5(a) depicts the distribution of a noise record of 600 samples produced by a noise generator with gaussian distribution. In Figure 5(b) the distribution of the above noise record after SVD equalization with ten singular values is illustrated. It is clear that the initial distribution is conserved.

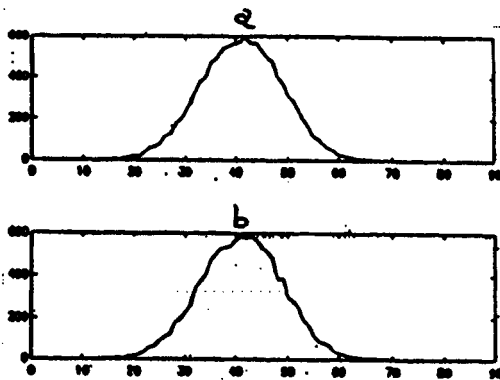


Figure 5. a) Initial noise distribution. 600 samples. b) Noise distribution after SVD equalization with ten singular values.

A new method has been proposed here for the improvement of the noise spectrum variance or else spectral flatness for short noise records. The key idea was that a "better noise" will have more equal its singular values. Consequently the decomposition of the noise will give a better control of the noise quality. The analysis by synthesis procedure proposed here can be useful and complement any software/hardware environment of noise generation and it proposed as a necessary utility. Further research is focused on the theoretical demonstration of the convergence to a noise of decreased variance.

## REFERENCES

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