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AN OPTIMAL INTEGER ALLOCATION SCHEME FOR SUB-BAND CODING OF SPEECH

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ABSTRACT

An optimal integer allocation algorithm for block quantization is introduced, which can be applied to Sub-Band Coding of Speech. The new scheme, which is a generalization of the various commonly used bit allocation (radix 2) schemes, improves current block quantization techniques in three aspects:

- Any integer number of quantization levels, rather than only radix 2 integers, can be assigned to the different blocks.
- The assigned levels are determined optimally using an efficient dynamic programming algorithm.
- The proposed algorithm can be used to generate optimal power of 2 integer levels as well, in contrast to some heuristic, sub-optimal algorithms which have been previously employed in bit allocation schemes.

This result in a better allocation of the bits resource to the different channels according to their respective energy. Thus, the total quantization noise is reduced and the quality of the reconstructed signal may improve appreciably.

INTRODUCTION

Block quantization is an efficient scheme to quantize a set of independently distributed random signals, using a different scalar quantizer for each of the individual signals. Thus, block quantization involves the allocation of the total available bits to the different signals, according to their statistical distribution.

Block quantization is widely used in Sub-Band Coding (SBC) of speech to code the different frequency bands (*ref* 1 – 4) , and in image transform coding to quantize the transform coefficients (*ref* 7) .

In SBC, the allocation of bits to the different frequency bands is done in an adaptive manner as to adjust the number of bits assigned to each band according to the instantaneous spectral density of the signal in this band. Since the spectral properties of speech are known to vary slowly, this adaptation process is carried out each time a new block of signal samples is being processed. An optimal allocation of bits to the different bands is based on minimizing the total quantization energy e of all the bands for a given number of bits (*ref* 2).

For each transmission block, denote by:

- b_i: The optimal number of bits assigned to the i-th band.
- E_i: The variance of the signal in the i-th band.

Then e can be expressed, to first order, as:

$$e = \sum_{i=1}^{N} \frac{E_i}{q_i^2} \qquad q_i = 2^{b_i} \qquad \text{s.t.} \quad \sum_{i=1}^{N} b_i = B \tag{1}$$

where B is the total available number of bits. In order to minimize the total quantization energy, e from Eqn. (1) should be minimized with respect to q_i . This minimization problem can be solved by use of a Lagrange multiplier, to yield (*ref* 2):

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$$b_i = \log_2 AE_i^{1/2} \quad ; \quad i = 1, \ldots, N$$

$$A = \frac{2^{B/N}}{\prod_{i=1}^{N} E_i^{1/2N}} \tag{2}$$

From Eqn. (2) it can be observed that the optimal number of bits assigned to the i-th band is logarithmically proportional to the energy of that band.

Following Eqn. (1), q_i can be interpreted as the decimal number of levels assigned to the i-th band. According to Eqn. (2) its optimal assignment is:

$$q_i = AE^{1/2} \quad ; \quad i = 1, \ldots, N$$

$$s.t. \quad \prod_{i=1}^{N} q_i = 2^B = Q \tag{3}$$

A more general treatment in $(ref\, 8 - 10)$ leads to the same optimal bit assignment.

Unfortunately, the optimal bit allocation cannot be implemented in practice since the analysis yields fractional bits b_i or correspondingly, decimal number of quantization levels q_i. Therefore, the bit values of the optimal allocation have to be rounded to the nearest integer, leading to a power of 2 number of quantization levels assigned to each band. Two disadvantages are associated with this procedure:

1. The total quantization energy of the bit allocation is greater than the optimal value, leading to a noticeable degradation in the quality of the reconstructed speech.
2. The algorithms used to obtain a rounded bit allocation which conforms to the constraint of Eqn. (1), are heuristic in nature and therefore sub-optimal.

This motivated the work outlined in this paper which offers a better allocation scheme for block quantization. In the following section, the novel optimal integer allocation scheme is presented. Then, a dynamic programming algorithm which is used to solve the optimal allocation problem, is outlined and finally, conclusions are drawn.

OPTIMAL INTEGER ALLOCATION

Denote by α_i the ratio between the optimal decimal allocation q_i and an integer allocation l_i for the i-th signal source (i-th sub-band):

$$\alpha_i = q_i / l_i \quad ; \quad i = 1, \ldots, N \tag{4}$$

By substituting the expression for q_i from Eqn. (3) in (4), Eqn. (1) for the total quantization energy can be rewritten for an arbitrary integer allocation l_i as follows:

$$e = \frac{1}{A^2} \sum_{i=1}^{N} \alpha_i^2$$

Hence, the optimal integer allocation problem can be casted, in a normalized form, as follows:

$$e\,A^2 = \min_{\alpha_i} \sum_{i=1}^{N} \alpha_i^2$$

$$s.t. \quad \prod_{i=1}^{N} \alpha_i \geq 1 \tag{5}$$

Note that the product constraint is equivalent to the constraint on the total number of decimal levels, expressed in Eqn. (3). The inequality stems from the fact that α_i, as defined in Eqn. (4), can have only discrete decimal values which correspond to the set of the integers l_i. Clearly, if α_i were not constrained, their optimal value would be exactly 1.

This formulation leads to an integer programming optimization problem which is solved by using a standard Dynamic Programming (DP) approach.

DYNAMIC PROGRAMMING ALGORITHM

The rationale of the algorithm is based on the fact that this global optimization problem can be decomposed into a series of N local optimization steps, as will be elaborated now. A partial quantization energy function $e(n,J)$ of the first n bands is defined as:

$$e(n,J) = \min_{\alpha_i} \sum_{i=1}^{n} \alpha_i^2$$
$$s.t. \quad \prod_{i=1}^{n} \alpha_i \geq J$$

where the range J of $e(n,J)$ is allowed to vary in some interval $[J_{\min},J_{\max}]$. This function may be partitioned as follows:

$$e(n,J) = \min_{\alpha_n}[\alpha_n^2 + e(n\text{-}1,J/\alpha_n)] \tag{6}$$

The dynamic programming algorithm then progress from $n=1$ to N, while at each stage a local optimization problem for the n-th band is solved by minimizing $e(n,J)$ with respect to the set of the allowable α_n for the n-th band only. Hence, the energy function is evaluated recursively until, at the end of the N-th stage, $e(N,1) = e\,A^2$ is the global minimum quantization energy solution. By back-tracking the solution, the optimal integer allocation l_i is obtained. The allowable set of α_n at the n-th stage is determined according to whether an odd (midtread), power of 2 or any integer allocation is desired.

The problem and its solution have a nice geometric interpretation which emerges from the definition of e and its associate inequality. For example, for $N=2$ it is clear that we look for the minimal circle which intersects a feasible point $[\alpha_1, \alpha_2]$ in the plane. For an N-dimensional space we seek for the minimal sphere which intersects a feasible point. Clearly, when l_i can take any decimal value, $\alpha_i = 1$ and the minimal sphere has a radius of exactly $\sqrt{N}$ and is tangent to an N-dimensional hyperbola which is defined by the product constraint on the α_i.

From the definition of $e(n,J)$ and its geometric interpretation it can be verified that $e(n,J)$ is a monotonic non-decreasing function in J. Moreover, since α_i are allowed to have only discrete decimal values, $e(n,J)$ is an ascending staircase (pure jumps) function. Therefore, at each stage n of the algorithm, $e(n,J)$ may be uniquely defined by the set of its magnitude at the jump points $\{J(n)\}$, over the range of $[J_{\min}(n),J_{\max}(n)]$. According to the DP recursion from Eqn. (6), the set $\{J(n)\}$ can be computed recursively as follows:

$$\{J(n)\} = \{J(n-1)\}*\{\alpha_n\}$$

where * denotes the product between the respective sets. The range of J may vary from stage to stage such that the search is narrowed according to the set of the candidate l_n (or α_n) at that stage (*ref* 11) .

The proposed algorithm may be used to determine an optimal bit allocation as well, by selecting α_n which correspond to power of 2 l_n integers. From the geometric interpretation it is also evident why bit allocation will yield quantization energy which is larger than that of the level allocation.

The final stage of the algorithm may yield several optimal integer allocation sequences which are permutations of each other. This situation may occur when some of the bands have the same energy level E_i . Therefore it is advisable to sort the bands according to some priority level (e.g. energy, frequency etc.).

CONCLUSIONS

An optimal integer allocation scheme for a block quantization was outlined. This scheme can be considered as a general problem of integer programming where a decimal vector q is to be converted to an integer vector l, such that the angle in the N-dimensional space between q and l is minimal. An efficient dynamic programming procedure was outlined to construct the integer vector l. There sts an alternative computational approach to bit allocation which is based on Marginal Analysis (*ref* 7). However, the proposed DP approach can be shown to be more efficient, especially when a candidate levels set, rather than candidate bits set, is to be searched.

From a comprehensive performance evaluation of integer and bit allocation schemes (*ref* 6), it was concluded that an appreciable quality improvement of the reconstructed speech is gained when integer rather than bit allocation is used.

Moreover, a statistical analysis of the speech samples at each sub-band reveals an exponential-like probability density function (PDF) which attains its maximum at the origin (*ref* 5). This result suggests that a midtread quantizer is preferred to a midriser one. Unfortunately, bit allocation schemes yield, by definition, a midriser quantizer whereas the proposed scheme can yield superior midtread quantizers as well.

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